SYLLABUS FOR M. PHIL IN MATHEMATICS

(Effective from the academic Session 2009 – 2010 and onwards)

THE UNIVERSITY OF BURDWAN

RAJBATI, BURDWAN

WEST BENGAL

THE UNIVERSITY OF BURDWAN

Syllabus for M. Phil in Mathematics

Duration of M. Phil course of studies in Mathematics will be Two years.

Course Structure: **Total Marks = 500**

First Year:

Paper-1 : **Research Methodology: 100 Marks**

Paper-II : **Optional Paper: 100 Marks** (Any one from the following two groups to be decided by the Department)

Applied Group

- Thermoelasticity and Magneto- thermoelasticity
- Turbulent Flows
- Advanced Optimization and Operations Research

Pure Group

- Functional Analysis with Applications
- Riemannian Geometry/Almost Complex Spaces
- Theory of Semigroups and Semirings

Paper-III :Term Paper & Seminar Presentation:100 Marks

- Term Paper-I(related to Paper-I): 25 Marks
- Term Paper-II(related to Paper-II): 25 Marks
- Seminar Presentation on Term Paper-I and Term Paper-II: 2x25= 50 marks

Second Year : **Dissertation: 200 marks**

- Written: 150 Marks & Viva Voce: 50 Marks

DETAILED SYLLABUS

M. Phil in Mathematics

Paper-1 : Research Methodology: 100 Marks

Unit-1: Research Methodology for Applied Mathematics: 50 Marks:

- Continuum Mechanics
- Optimization
- Methods of Applied Mathematics
- Computational Methods

Detailed Syllabus on Paper-I (Unit -1)

Continuum Mechanics (Marks: 15)

Principles of continuum mechanics, Cauchy stress equations of motion, energy balance, stress-strain relations for isotropic elastic medium, stress-strain rate relations in linearly homogeneous isotropic fluid media, Navier-Stokes equations of motion, Helmholtz's vorticity equation (viscous fluid). (15 L)

Optimization (Marks: 10)

Direct and gradient based methods for constrained and unconstrained optimization problems. (10 L)

Methods of Applied Mathematics (Marks: 15)

Fundamental properties of eigen values and eigen functions for symmetric kernels, Hilbert Schmidt theorem and some immediate consequences, solutions of integral equations with symmetric kernels. (15 L)

Computational Methods (Marks: 10)

Numerical solution of linear and nonlinear ordinary differential equations, numerical solution of linear partial differential equations. (10 L)

References :

1. Leigh, D.C., Non-linear Continuum Mechanics, MGH.

2. Eringen, A.C., Non-linear Theory of Continuous Media, Academic Press, 1962.
3. Soholnikoff, I. S., Mathematical Theory of Elasticity, MGH.
4. Chandrasekhariah, D. S. and Debnath, L., Continuum Mechanics, Academic Press.
5. Chang Edwin, K.P. and Zak, S., An Introduction to Optimization, John Wiley & Sons Inc., 2004.
6. Aokie, M., Introduction to Optimization Techniques : Fundamentals and Applications of Nonlinear Programming, Macnillan, 1971.
7. Sun, W. and Yuan, Y., Optimization Theory and Methods : Nonlinear Programming, Springer, 2006.
8. Bazaraa, M.S., Shreali, H.D. and Shetty, C.M., Nonlinear programming : Theory and Algorithms, John Wiley & Sons., 2004.
9. Tricomi, F.G., Integral Equations, Dover Pub., 1985.
10. Kress, R., Linear Integral Equations, Springer-Verlag, 1999.
11. Mikhlin, S.G., Integral Equations, Pergaman Press, 1964.
12. Hochstadt, H., Integral Equations, Wiley, 1973.
13. Numerical Methods for Mathematics, Science and Engineering – J. W. Mathews-PHI.
14. Introductory Methods of Numerical Analysis – S. S. Sastry – PHI.
15. Numerical Solution of Partial Differential Equations – G. D. Smith.

Unit-2: Research Methodology for Pure Mathematics: 50 Marks:

- Basics of Functional Analysis , Real Analysis and Topology
- Ring Theory
- Tensor Calculus with Applications
- Fundamentals of Riemannian Geometry

Detailed Syllabus on Paper-I (Unit -2)

Basics of Functional Analysis (**Marks: 5**)

Review of basic properties of Metric Spaces, Banach spaces, Hilbert Spaces and Topological Vector Spaces. (5 L)

References :

1. W. Rudin, Functional Analysis, Tata Mc Graw Hill Pub. Co., New Delhi, 1974.
2. E. Kreyszig, Introductory Functional Analysis with applications, John Wilet & Sons, 1978.
3. G. Bachman and L. Narici, Functional Analysis, Academic Press,1966
4. B. K. Lahiri, Elements of Functional Analysis, The world Press Pvt. Ltd. Kolkata, 1994.
5. C. Goffman and G. Pedrick, First course in Functional analysis, Prentice hall, 1974.

Real Analysis (Marks:4)

Measure, measurable space, measurable functions, Integral of a function with respect to a measure. (4 L)

References :

1. S. K. Berberian, measure and integration, Chelsea publishing Company, Bronx, New York.
2. P. R. Halmos, measure theory, Van nostrand, New York, 1950.

Topology (Marks: 5)

Review of topological spaces, separation axioms, countability,separability, compactness, locally compactness, connectedness, locally connectedness, metrizability, Urysohn's metrization theorem. (5 L)

References :

1. James Dugundji, Topology, prentice hall of India Pvt. Ltd. 1975.
2. J.L. Kelly General topology Spinger verlag, 1961.
3. James Munkres, topology Pearson Education Asia, 2001
4. M.G. Murdreswar, general topology, Wiley Eastern Ltd. 1983.
5. G. F. Simmons, Introduction to topology and modern Analysis, Mc Graw-Hill Book Company, 1963.

Ring Theory (**Marks: 12**)

Radical, Jacobson radical, prime radical, prime ideal and m-system, prime ring, semi-prime ideal and n-system, semi-prime ring, direct product and subdirect product of rings, subdirectly irreducible rings, Boolean rings. (12 L)

References :

1. N. H. McCoy; The Theory of Rings, Chelsea Publishing Company, Bronx, New York, 1973.
2. D. M. Burton; A First Course in Rings and Ideals, Addison-Wesley Publishing Company, London, 1970.

Tensor Calculus (Marks: 12)

Tensor as a generalization of vector, Addition and subtraction of tensors, Symmetric and skew symmetric tensors, Invariant, Quotient law, Outer multiplication, Inner product, Christoffel symbols, Covariant derivatives (12 L)

Fundamentals of Riemannian Geometry (Marks: 12)

Riemannian space with examples, Riemannian curvature tensor and its properties, Ricci tensor, Scalar curvature, Ricci's Theorem, Geodesic (12 L)

References :

1. B. Spain, Tensor calculus, A concise course, Dover Publication, 1965.
2. I. S. Sokolnikoff, Tensor Analysis, Theory and application to geometry and mechanics of continua. John Wiley, (Second edition), 1964.
3. U. C. De, A. A. Shaikh and J. Sengupta, Tensor calculus, (second edition), Narosa Publishing House, Pvt, Ltd. 2007.

Paper-II : **Optional Paper: 100 Marks** (Any one from the following two groups to be decided by the Department)

Applied Group

1. Thermoelasticity and Magneto- thermoelasticity
2. Turbulent Flows
3. Advanced Optimization and Operations Research

Pure Group

1. Functional Analysis with Applications
2. Riemannian Geometry/Almost Complex Spaces
3. Theory of Semigroups and Semirings

Detailed Syllabus on Paper-II : Optional Paper (Applied Group)

1. Thermo elasticity and Magneto – Thermo elasticity Total Lectures: 100

Thermoelasticity : Constitutive equations of thermoelasticity. Energy equation. Coupled parabolic heat transport equation. Displacement equation of motion. Basic equations of dynamic coupled thermoelasticity. Non-dimensional form. Generalized thermoelasticity (statement only). Wave-type heat transport equation (No deduction). Basic equations. Propagation of plane waves in a thermoelastic medium (one-dimensional deformation). Dispersion equation. (50 L)

Magneto-thermoelasticity : Fundamentals of Magnetoelasticity. Fundamentals of magneto-thermoelasticity. Interaction among electromagnetic, thermal and mechanical fields. Basic equations (Statement only). Displacement equations of motion. Coupled parabolic heat transport equation. Linearised equations in Magneto-thermoelasticity. Propagation of Magneto-thermoelastic plane waves (one-dimensional deformation). Case of infinite electrical conductivity. Generalizations of the theory (Statement only) (50 L)

References *:*

1. Thermoelasticity – W. Nowacki, Addison – Wesley Pub. Comp., Inc.
2. Advances in Applied Mechanics, vol. 10, Academic Press, 1976.
3. Theory of Thermoelasticity with applications – J.L.Nowinsky

4. Thermoelasticity – H. Parkus – 1968, Waltham, Mass Blaisdell, 99 – 100.
5. Dynamic Coupled Thermoelasticity – Dhaliwal and Singh, Hindustan Pub. Corporation, India.

2. Turbulent Flows

Introduction, origin of turbulent, mean and fluctuation motions, Reynold's equation for mean motion turbulent flow, Reynold's stress-tensor. (20 L)

Eddy viscosity and semi-empirical theories of turbulence, Prandtl's mixing length theory, momentum transfer theory, Taylor's vorticity transfer theory, Karman's similarity hypothesis. (20 L)

The Universal velocity profile near a wall, turbulent flow in smooth and rough pipes, velocity distribution in channel flow under constant pressure gradient, structure of the turbulent boundary layer, turbulent boundary layer over a smooth flat plate, Log law and Power law profiles, turbulent wake behind a symmetrical cylinder. (30 L)

Statistical approach : Introductory concepts, double correlation between velocity components, longitudinal and lateral correlations in homogeneous turbulence, Eulerian correlation with respect to time, Taylor's one-dimensional energy spectrum, energy relations in turbulent flows. (30 L)

References :

1. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New Edition, 2006.
2. S. W. Yuan, Foundations of Fluid mechanics, 1969.
3. J. Hinze, Turbulence, 2nd edn. McGraw Hill, New York, 1975.
4. Batchelor, The Theory of Homogeneous Turbulence, Cambridge University Press, 1955.
5. D. J. Tritton, Physical Fluid Dynamics, 2nd edn., Oxford Science Publications, 2005.

3. Advanced Optimization and Operations Research

Differentiable convex and concave functions, optimality criteria in nonlinear programming with differentiability, duality in nonlinear programming, generalization of convex function, quasi convex, strictly quasi convex and pseudo convex functions. (25 L)

Penalty function technique, interior and exterior penalty function techniques. (6 L)

Stochastic programming and its computational aspects. (4 L)

System reliability of different systems, reliability optimization. (20 L)

Inventory models with fixed and imprecise parameters. (15 L)

Real coded genetic algorithms and applications. (15 L)

Decision making problems with interval valued parameters. (15 L)

References :

1. Bazaraa, M. S., Sherali, H. D. and Shetty, C. M., Nonlinear Programming, John Wiley & Sons. Inc., 2004.
2. Mangasarian, O. L., Meyer, R. R. and Johnson (Eds.) Nonlinear Programming, Academic Press, New York, 1975.
3. Michalawicz, Z., Genetic Algorithms + Data Structure = Evaluation Programs, Berlin: Springer Verlag, 1996.
4. Hansen, E. and Walster, G. W., Global Optimization Using Interval Analysis, Marcel Dekker Inc., 2004.
5. Rao, S. S., Optimization theory and Applications, Wiley Eastern Ltd., New Delhi.
6. Chang, Edwin, K. P. and Zak, Stanislaw, An Introduction to Optimization, John Wiley & Sons Inc., 2004.

Detailed Syllabus on Paper-II : Optional Paper (Pure Group)

1. Functional Analysis with Applications **Total lectures 100**

Functions from R^n to R^m, continuity, differentiation, partial derivatives, derivatives, inverse functions, implicit functions; integration, integrable functions,Fubini's Theorem, partitions of unity. (25L)

Spectral properties of bounded linear operators, compact operators on normed linear spaces, spectral properties of compact operators on normed linear spaces, operator equations involving compact operators, Fredholm alternative. (25L)

Reflexivity, uniform convexity and some general theorems, Helley's theorem, Milman Pettit's theorem, Eberlein's theorem. (10L)

Fixed point theory and its applications to integral equations, approximations in normed linear spaces, uniform approximation, and approximation in Hilbert spaces. (15L)

Random operators, random fixed point equation, non expansive mappings and its applications to fixed point theory. (25L)

References :

1. M. Spivak: *Calculus on Manifolds*, The Benjamin/Cummings Publishing Company, 1965.
2. Brown and Page: *Elements of Functional Analysis*, Von Nostrand Reinhold Company, 1970.
3. Courant, *Differential and Integral Calculus*, Vol-II, Interscience, New York,1937.
4. Burkil & Burkil: *Theory of functions of a real variable*.
5. J.R. Munkers: *Analysis on Manifolds*, Wiley Eastern Ltd.
6. W. Flemming: *Functions of several variables*
7. S. Lang: *Functions of several variables*.
8. N.L. Akhiezer and I.M. Glazman, Theory of Linear operators in Hilbert spaces, Vol.1, Frederick Ungar Pub. Co. 1966.
9. A.V. Balakrishnan, Applied Functional analysis, Springer-Verlag, 1976.
10. L. Debnath and P. Mikusin'ski, Introduction to Hilbert spaces with applications, Academic Press, 1990.
11. M.C. Joshi and R.K. Bose, Some topics in nonlinear functional analysis, wiley Eastern Ltd., 1985.
12. A.N. Kolmogorov and S.V. Fomin, Elements of the theory functions and functional Analysis, Nauka, Moscow, 1989.
13. E. Kreyszig, Introductory Functional Analysis with applications, John Wilet& Sons, 1978.
14. G.Bachman and L. Narici, Functional Analysis, Academic Press, 1966.
15. K. Yosida, Functional Analysis, Springer Verlag, 1980.
16. Dunford and Schwartz, Linear operators, (Vol.1& II), Interscience, New York, 1963.
17. L.V. Kantorvich and G.H. Akilov, Functional Analysis, Pergamon Press, 1982.
18. B.K. Lahiri, Elements of Functional Analysis, The world Press Pvt. Ltd. Kolkata, 1994.
19. A.L. Brown and A. Page, Elements of Functional Analysis, Van Nostrand Reinhold Co., 1970.
20. K. Goebel and W.A. Kirk, Topics in Metric fixed point Theory, Cambridge University Press, 1990.
21. J.P. Aubin, Applied Abstract Analysis, John Wiley & Sons, 1977.
22. C.L. De Vito, Functional Analysis and Linear operator theory, Addison Wesley Pub. Co., 1990.

23. A.N. Kolmogorov and S.V. Fomin, Functional Analysis, Vols I & Ii, Graylock Press, Rochester, 1957.
24. W. Rudin, Functional Analysis, Tata Mc Graw Hill Pub. Co., New Delhi, 1974.
25. J.B. conway, A course on Functional Analysis, Springer Verlag, 2nd Edition, 1990.

2. Riemannian Geometry **Total lectures 100**

Differentiable manifolds (definition and examples). Tangent space and cotangent space (definition only). (10 L).

Vector fields, Lie bracket, Lie group and Lie algebra (definition and examples), exterior product and exterior derivative (definition and examples only). (15 L).

Linear connection (definition and examples), curvature and torsion of a linear connection, covariant derivative. (10 L).

Riemannian manifolds, properties of Riemannian curvature tensor, Koszul formula, Riemannian connection, Fundamental Theorem of Riemannian geometry, Riemannian manifold as a metric space. (20 L).

Sectional curvature, Schur's Theorem, Ricci tensor, scalar curvature, Einstein manifold. Semisymmetric metric connection. (20 L).

Killing vector field, conformal Killing vector field, gradient vector field, Conformal curvature tensor, projective curvature tensor. (25 L).

References :

1. K. Yano and M. Kon, Structure of manifolds, World Scientific Publishing Co. Pvt. Ltd. 1984.
2. R. S. Mishra, Structure on a differentiable manifold and their applications. Chandrama Prakashan, Allahabad 1984.
3. U. C. De and A. A. Shaikh, Differential geometry of manifolds, Narosa publishing House Pvt. Ltd., 2007.

2. Almost Complex Spaces **Total lectures 100**

Definition and examples of differentiable manifolds, Tangent spaces, Lie derivatives, Wedge product, exterior derivative (definition and example). Definition and examples of Riemannian manifolds, Linear connections and Riemannian connections (40 L).

Almost complex manifolds, Nijenhuis torsion tensor, Almost Hermite manifolds, Almost Analytic vector fields, Curvature tensor, Linear connections, Kaehlerian manifolds, Nearly Kaehlerian manifolds, Almost Kaehlerian manifolds, Curvature identities, Holomorphic sectional curvature, Application of Kaehlerian manifolds (60 L).

References :

1. K. Yano and M. Kon, Structure of manifolds, World Scientific Publishing Co. Pvt. Ltd. 1984.
2. R. S. Mishra, Structure on a differentiable manifold and their applications. Chandrama Prakashan, Allahabad 1984.
3. U. C. De and A. A. Shaikh, Complex manifolds and Contact manifolds, Narosa Publishing House Pvt. Ltd., 2007.

3. Theory of Semigroups and Semirings **Total lectures 100**

Semigroup: Definitions, examples, semilattice, congruences, ideals and Rees congruences, lattice of congruences, Green's relation, regular semigroup, inverse semigroup, simple and 0-simple semigroup, primitive idempotents, union of groups, semilattices of groups, bands, orthodox semigroup. (60 L)

Semiring: Definition, examples, prime and semiprime ideals in semirings, quotient semirings, morphism of semirings, kernel of morphisms, semirings of functions, additively regular semiring, Euclidean semiring and completely regular semiring, semiring with some conditions. (40 L)

References :

1. J. M. Howie; An Introduction to Semigroup Theory, Academic Press, London, 1976.
2. J. S. Golan; Semiring and Their Applications, Kluwer academic publishers, Boston, 1999.
3. U. Hebisch & H. J. Weinert; Semirings, World Scientific, Singapore, 1993.